

# The 9th East Asia Number Theory Conference

## Titles and Abstracts

JST (UTC+9)	23 (Mon)	24 (Tue)	25 (Wed)	26 (Thu)	27 (Fri)
10:30 - 11:20	M. Kurihara	D. Wei	H. Yoo	S. Lim	W. Kuo
11:40 - 12:30	M. Yu	T. Takamatsu	R. Zhang	P.-J. Wong	D. Kim
15:00 - 15:50	P. Xi	J. Guo	Y. Okumura	M. Oi	J. Fang
16:10 - 17:00	R. Sugiyama	J. Lee	Y.-L. Kuan	S.-Y. Chen	L. Fu

### Monday August 23, 2021

10:30 - 11:20

Masato Kurihara (Keio University)

Title: Some analytic quantities in the arithmetic of elliptic curves

Abstract: For a rational elliptic curve, we introduce and discuss some quantities defined analytically, giving information on the Mordell-Weil rank and on Selmer groups, in a form different from the conjecture of Birch and Swinnerton-Dyer. We explain that these analytic quantities control the maps of reduction mod primes on the Mordell Weil group and Selmer groups.

---

11:40 - 12:30

Myungjun Yu (KIAS)

Title: Bounds for 2-Selmer ranks in terms of a modified ideal class group

Abstract: Let  $E$  be an elliptic curve over the rational number field  $\mathbb{Q}$ . Selmer Groups and Ideal Class Groups are important and widely-studied objects in number theory. Brumer and Kramer studied relations between these two objects in their paper in 1977. They actually found an upper bound for the 2-Selmer rank of  $E$  in terms of the ideal class group of a certain cubic field extension of  $\mathbb{Q}$ . As an application, they determined the Mordell-Weil ranks of (most) elliptic curves of prime conductor assuming the BSD conjecture. In this talk, we will talk about a generalization of Brumer-Kramer's work to the case of elliptic curves over an arbitrary number field. We will give both upper and lower bounds for the 2-Selmer rank in terms of a (modified) ideal class group, and the bounds turn out to be sharp in many cases. This is joint work with Hwajong Yoo.

---

15:00 - 15:50

Ping Xi (Xi'an Jiaotong University)

Title: Lang-Trotter conjecture for CM elliptic curves

Abstract: For any elliptic curve  $E$  over  $\mathbf{Q}$  and any non-zero integer  $r$ , the Lang-Trotter conjecture has predicted the asymptotic behaviours of the number of good primes  $p \leq x$ , denoted by  $\pi_{E,r}(x)$ , such that the Frobenius trace of  $E$  at  $p$  is equal to the given integer  $r$ . Quite recently, we are able to prove an estimate for  $\pi_{E,r}(x)$  which confirms the upper bound part of the conjecture for CM elliptic curves. Moreover, intimate connections of this conjecture and Hardy-Littlewood conjecture can also be established to characterize the shape of the Lang-Trotter constant in  $\pi_{E,r}(x)$ . This is based on the joint work with Daqing Wan (in progress).

---

16:10 - 17:00

Rin Sugiayama (Japan Women's University)

Title: Tensor structures of modulus sheaves with transfers

Abstract: Towards to a generalization of Voevodsky's category of motives, Kahn-Saito-Yamazaki and Kahn-Miyazaki-Saito-Yamazaki have introduced modulus sheaves with transfers, its cube-invariance and reciprocity sheaves with transfers. The tensor product of homotopy invariant sheaves with transfers is generalized to reciprocity sheaves via the theory of cube-invariant modulus sheaves. I will talk about some properties of the tensor structures. In particular, I will show new motivic presentations of the absolute Kähler differentials and the first infinitesimal neighborhood of the diagonal. This is a joint work with Kay Rülling and Takao Yamazaki.

---

## Tuesday August 23, 2021

10:30 - 11:20

Dasheng Wei (Academy of Mathematical and System Science)

Title: Rational points on fibration with few non-split fibers

Abstract: Let  $f : X \rightarrow \mathbb{P}^1$  be a dominant map whose generic fibre is rationally connected. Assume that the Brauer-Manin obstruction controls Hasse principle and weak approximation for rational points on all (or most) smooth fibers. A natural question is whether the same holds for the whole space  $X$ . With some assumptions, we will try to answer this question and give some applications. This is a joint work with Harpaz and Wittenberg.

---

11:40 - 12:30

Tepei Takamatsu (The University of Tokyo)

Title: Minimal model program for semi-stable threefolds in mixed characteristic

Abstract: The minimal model program, which is a higher-dimensional analog of the classification method of surfaces, is a tool to find a "simplest" variety in each birational equivalence class. The minimal model program is also studied for more general schemes not necessarily defined over a field, and play an important role in studies of reductions of varieties. Kawamata showed that the minimal model program holds for strictly semi-stable schemes over an excellent Dedekind scheme of relative dimension two whose each residue characteristic is neither 2 nor 3. In this talk, I will introduce a generalization of the result of Kawamata without any assumption on the residue characteristic. This talk is based on a joint work with Shou Yoshikawa.

---

15:00 - 15:50

Ji Guo (Academia Sinica)

Title: A Truncated Second Main Theorem for Algebraic Tori with Moving Targets and Applications

Abstract: In this talk, we will establish a second main theorem for algebraic tori with moving targets with truncation to level 1. Then we will apply it to a special case of Green-Griffiths-Lang conjecture and the integrability of the ring of exponential polynomials in the ring of entire functions. We will also discuss the connections with Vojta's conjecture and linear recurrences over number fields. This is a joint work with Chia-Liang Sun and Julie Wang.

---

16:10 -17:00

Jungin Lee (KIAS)

Title: Counting algebraic tori over  $\mathbb{Q}$  by Artin conductor

Abstract: Counting number fields by discriminant is one of the most important topics in arithmetic statistics. In this talk, we discuss its natural generalization: counting algebraic tori over  $\mathbb{Q}$  of given dimension by Artin conductor. We propose analogues of Linnik's and Malle's conjecture for tori over  $\mathbb{Q}$  and provide several evidences for them. After that, we summarize our results on counting two and three-dimensional tori over  $\mathbb{Q}$ .

---

### Wednesday August 23, 2021

10:30 - 11:20

Hwajong Yoo (Seoul National University)

Title: The rational torsion subgroup of  $J_0(N)$

Abstract: In this talk, we review the work of Mazur and Ohta about the rational torsion subgroup of  $J_0(N)$ , and discuss a natural generalization of their work.

---

11:40 - 12:30

Runlin Zhang (Beijing International Center for Mathematical Research, Peking University)

Title: Counting integral points on affine homogeneous varieties and homogeneous dynamics

Abstract: Given an affine variety over the integers, assuming there are infinitely many solutions, it is natural to ask what the asymptotic count of the solutions is. It has been observed that when the variety is homogeneous, this problem is naturally related to a problem from homogeneous dynamics. I will report on progress on this problem and what remains to be done.

---

15:00 - 15:50

Yoshiaki Okumura (Toyo University)

Title: On congruence of Galois representations attached to  $A$ -motives

Abstract: The notion of  $A$ -motives is a higher-dimensional generalization of Drinfeld modules, and  $\mathfrak{p}$ -adic representations attached to them are one of the most important objects in function field arithmetic. In this talk, we give a criterion for two congruent  $\mathfrak{p}$ -adic representations arising from  $A$ -motives defined over a global function field to be isomorphic up to semi-simplification when they are restricted to a decomposition group, in terms of Hodge-Pink theory. This is a function field analogue of the criterion for  $\ell$ -adic representations given by Ozeki and Taguchi.

---

16:10 - 17:00

Yen-Liang Kuan (National Dong Hwa University)

Title: Linear Relations among Triple Multiple Zeta Values

Abstract: Multiple zeta values in the positive characteristic were initially studied by Thakur, who defined them as analogues of classical multiple zeta values of Euler. In this talk, we will give necessary conditions for linear relations among triple multiple zeta values of same weights.

---

**Thursday August 23, 2021**

10:30 - 11:20

Subong Lim (Sungkyunkwan University)

Title: Independence between coefficients of two modular forms

Abstract: A modular form is determined by its Fourier coefficients. A natural question is to find relations between two modular forms when a connection between their Fourier coefficients is given. Suppose that  $f$  and  $g$  are nonzero weakly holomorphic modular forms on  $SL(2, \mathbb{Z})$ . We prove that if the number of ratios of Fourier coefficients of  $f$  and  $g$  are finite, then  $f$  is a constant multiple of  $g$ .

---

11:40 - 12:30

Peng-Jie Wong (NCTS)

Title: Refinements of strong multiplicity one for  $GL(2)$

Abstract: Let  $f_1$  and  $f_2$  be (holomorphic) newforms of same weight and with same nebentypus, and let  $a_{f_1}(n)$  and  $a_{f_2}(n)$  denote the normalised Fourier coefficients of  $f_1$  and  $f_2$ , respectively. If  $a_{f_1}(p) = a_{f_2}(p)$  for almost all primes  $p$ , then it follows from the strong multiplicity one theorem that  $f_1$  and  $f_2$  are equivalent. Furthermore, a result of Ramakrishnan states that if  $a_{f_1}(p)^2 = a_{f_2}(p)^2$  outside a set of primes  $p$  of density less than  $\frac{1}{18}$ , then  $f_1$  and  $f_2$  are twist-equivalent. In this talk, we will discuss some refinements of the strong multiplicity one theorem and Ramakrishnan's result for general  $GL(2)$ -forms. In particular, we will analyse the density of primes  $p$  for which  $|a_{f_1}(p)| \neq |a_{f_2}(p)|$  for non-twist-equivalent  $f_1$  and  $f_2$ .

---

15:00 - 15:50

Masao Oi (Kyoto University)

Title: Comparison of algebraic and geometric constructions of supercuspidal representations

Abstract: Supercuspidal representations are building blocks in representation theory of  $p$ -adic reductive groups. In this talk, I will report a result on comparing two different constructions of supercuspidal representations; one is an algebraic construction due to Yu, and the other one is a geometric construction due to Chan–Ivanov. I am also going to discuss how interesting this comparison result is from a viewpoint of the local Langlands correspondence. This is joint work with Charlotte Chan.

---

16:10 - 17:00

Shih-Yu Chen (Academia Sinica)

Title: On Deligne's conjecture for symmetric sixth  $L$ -functions of modular forms

Abstract: In this talk, we present our recent result on Deligne's conjecture for symmetric sixth  $L$ -functions of modular forms. We define automorphic periods associated to globally generic cuspidal automorphic representations of  $GSp(4)$  and show that the algebraicity of critical  $L$ -values for  $GSp(4) \times GL(2)$  can be expressed in terms of these periods. In the case of Kim-Ramakrishnan-Shahidi lifts of  $GL(2)$ , we establish period relations for the automorphic periods and powers of Petersson norm of modular forms. The conjecture for symmetric sixth  $L$ -functions then follows from our previous work on the algebraicity of adjoint  $L$ -functions for  $GSp(4)$ .

---

## Friday August 23, 2021

10:30 - 11:20

Wentang Kuo (The University of Waterloo)

Title: On multi-dimensional Hilbert-Kamke's problem

Abstract: Motivated by Lagrange's theorem that every natural number is the sum of at most four squares, in 1770, Waring asked whether each natural number  $k$  has an associated positive integer  $s$  such that every natural number is the sum of at most  $s$  natural numbers raised to the power  $k$ . Then, in 1900, Hilbert asked the solubility for a system of Diophantine equations of Waring type, now called the Hilbert-Kamke system. A multi-dimensional analogue of those problems is naturally raised by solving the same system in the polynomial rings over integers. A way to approach those questions is via the Hardy-Littlewood circle method. To ensure that the major arc from the circle method provides the main contribution, one needs to show an absolute positive lower bound of the singular series, a product of the densities of local solutions. This talk will discuss this lower bound problem for multi-dimensional Hilbert-Kamke's system by studying its local solubility. The number of variables we need is near-optimal. It is joint work with Yu-Ru Liu and Xiaomei Zhao.

---

11:40 - 12:30

Dohyeong Kim (Seoul National University)

Title: Integral unipotent completion and arithmetic applications

Abstract: Unipotent completion is defined over a field of characteristic zero. In this talk, we show how to obtain a  $p$ -integral generalization of it based on Lazard's theory. As an application one can define discrete Selmer sets, a notion introduced by Sakugawa, in greater generality.

---

15:00 - 15:50

Jiangxue Fang (Capital Normal University)

Title: Composition series for GKZ-systems

Abstract: In the 1980s, Gelfand, Kapranov and Zelevinsky generalized the hypergeometric functions in a unified way. They associated to any matrix  $A$  with integer entries a class of systems of partial differential equations or  $D$ -modules, which are now called GKZ-systems or  $A$ -hypergeometric  $D$ -modules. In this talk, I will review some basic results of the GKZ-systems. Especially, I will give a filtration on the GKZ-systems with semisimple subquotients using toric varieties.

---

16:10 - 17:00

Lei Fu (Tsinghua University)

Title: On the  $p$ -adic Gelfand-Kapranov-Zelevinsky hypergeometric system

Abstract: Using the Dwork theory, we construct the  $p$ -adic GKZ hypergeometric system, which is a twisted de Rham complex for a rigid analytic space. The underlying  $D$ -module corresponds to the GKZ hypergeometric system of differential equations. The underlying Frobenius structure gives an expression for the GKZ hypergeometric exponential sum.

---